Structural Analysis

For

Civil Engineering

By

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Syllabus for Structural Analysis

Statically Determinate and Indeterminate Structures by Force/ Energy Methods; Method of Superposition; Analysis of Trusses, Arches, Beams, Cables and Frames; Displacement Methods: Slope Deflection and Moment Distribution Methods; Influence Lines; Stiffness and Flexibility Methods of Structural Analysis.

Previous Year GATE Papers and Analysis

GATE Papers with answer key

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Subject wise Weightage Analysis

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"The starting point of all achievement is DESIRE. Keep this constantly in mind. Weak desire brings weak results, just as a small fire makes a small amount of heat."

.... Napoleon Hill

CHAPTER 1 Introduction to Structures

Learning Objectives

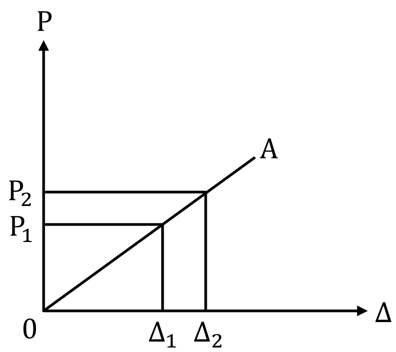
After reading this chapter, you will know:

1. Fundamental Assumptions
2. Classification of Structures
3. Classification of Skeletal Structures
4. Poission's Ratio
5. Bernoulli's Assumption In Structural Analysis

Introduction to Structures

1. **Structure (Def.):** When any elastic body is subjected to a system of loads and deformation take space and the resistance is setup against the deformation, then the elastic body is known as 'structure' if no resistance is set up in the body against the deformation, it is known as a 'Mechanism'.

2. **Fundamental Assumptions:**
   - A body is assumed to be Elastic.
   - A body is assumed to be homogeneous, if a body has identical properties in identical directions at all point of it, it is known as 'Homogeneous'.
   - A body is assumed to be isotropic. If a body has identical properties in all directions at a point, it is known as isotropic.
   - A body is assumed to be solid continuous structure.
   - Principle of super position is valid.
     According to this principle, the response of a structure on account of the combined action of any two systems of external forces 'P_1' and 'P_2' is equal to the sum of the responses due to the two systems of forces acting separately.



i.e., $\Delta_{(1+2)} = \Delta_1 + \Delta_2$

Where,

$\Delta_{(1+2)}$ = Total displacement due to the combined action of P_1 and P_2 applied in sequence of P_1 and P_2

Δ_1 = Displacement due to the action of P_1 alone

Δ_2 = Displacement due to the action of P_2 alone

Validity of Superposition Principle:

It is valid when

- The structure is in a condition of static equilibrium.
- The material of the structure behaves linearly (i.e., load versus deformation variation is a straight line)
- The supports are unyielding.
- Not valid for the slender columns.

3. **Classification of Structures:**
   - Skeletal Structures: Structures which can be idealized to a series of straight or curved lines. **E.g.,**: Roof trusses, building frames
   - Surface Structures: Structures which can be idealized to plane or curved surfaces. **E.g.,**: Slabs and shells
   - Solid Structures: Structures which can neither be idealized to a skeleton nor to a plane or curved surface. **E.g.,**: Massive foundation

4. **Classification of Skeletal Structures:**

   i. (a) **Pin Jointed Frames** (b) **Rigid Jointed Frames**

   (a) **Pin Jointed Frames:** Members are connected by means of pin joints. These frames support the loads by developing only axial forces, if the external loads act at the joints and members are straight.

   (b) **Rigid Jointed Frames:** Assumptions: The joints of rigid jointed frames are assumed to be rigid so that the angles between the members meeting a joint remain unchanged.
   These frames resist external forces by developing bending moments, shear forces, axial forces and twisting moments in the members of the frame.

   ii. (a) **Plane Frames** (b) **Space Frames**

   (a) **Plane Frames:** All members of the plane frames as well as the external loads are assumed to be in one frame.

Further Classifications:

Pin jointed plane frame: Members carry only axial forces.

Rigid jointed plane frame: Members are subjected to axial forces, shear forces and bending moments.

Important Note:

- If loaded in its own plane, any cross-section of member is subjected to three internal forces (one axial force, one shear force and one bending moment).
- If loading is away from the plane torsional moments also.

(b) **Space Frames:** All members do not lie in one plane. Very often, it is also a combination of series of frames.

Further Classifications
Pin jointed space frame: Members are subjected to axial forces only.
Rigid jointed space frame: Members are subjected to axial forces, shear forces, bending moments and twisting moments.

Important Note: Any cross section of a member of a skeletal space structure, there are six internal force components Viz., one axial force, biaxial shear force components Q_x and Q_y twisting moments 'T' and biaxial bending moments 'M_x' and 'M_y'.

5. **Poission's Ratio (μ or $1/m$):**
Defined as the ratio of lateral strain to linear strain
General values: Concrete = 0.15, Steel: 0.33, Cork = 0, Isotropic materials = 0.25
Note: For ideal elastic in-compressible materials Poission's ration is maximum of 0.5
(E.g., Rubber)

6. **Bernoulli's Assumption in Structural Analysis:**
"Plane sections which are normal to the neutral axis before bending remain plane and normal to the neutral axis after bending".
It leads to linearly varying strain over the cross-section.
Validity:
- Valid for elastic (working stress), limit state and ultimate (plastic) theories.
- Valid for prismatic or non-prismatic members.
- Valid for shallow beams only.
- Not valid for deep beams and locations of high shears.

7. **Equations of Static Equilibrium:**
Using the Cartesian system of coordinates as the reference frame, the equations of static equilibrium may be written as
$\Sigma F_x = \Sigma F_y = \Sigma F_z = 0$
$\Sigma M_x = \Sigma M_y = \Sigma M_z = 0$
Where, $\Sigma F_x, \Sigma F_y$ and ΣF_z are algebraic sums of the components of all external forces including reactive forces, along $x-, y-$ and $z-$ axis respectively and $\Sigma M_x, \Sigma M_y$ and ΣM_z are the algebraic sums of the moments of all external forces, including reactive forces, about $x-, y-$ and $z-$ axis respectively.

“Success is getting what you want.
Happiness is wanting what you.”

..... **Dale Carnegie**

CHAPTER 2 Trusses and Arches, Cables

Learning Objectives

After reading this chapter, you will know:

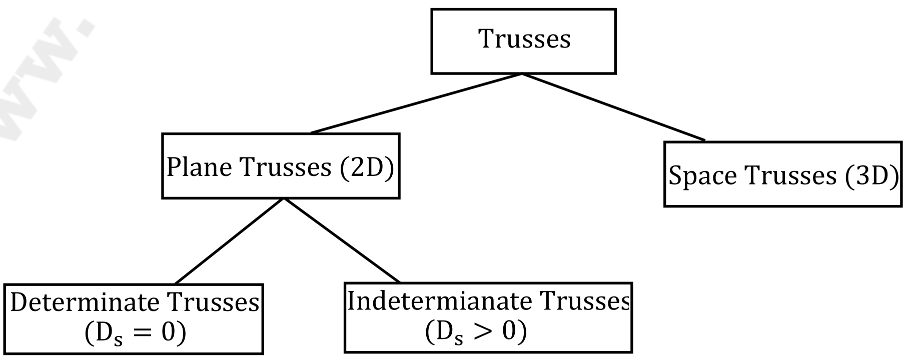
1. Trusses
2. Method of Sections
3. Zero Force Member
4. Temperature
5. Arches, Temperature Effect on Two Hinged Arches
6. Cables

Trusses

Trusses are statically determinate when all the bar forces can be determined from the equations of statics alone. Otherwise the truss is Statically Indeterminate. A truss may be statically/externally determinate or indeterminate with respect to the reactions (more than 3 or 6 reactions in 2D or 3D problems respectively). A truss may be internally determinate or indeterminate.
If we refer to j as the number of joints, R the number of reactions and M the number of members, than we would have a total of m + R unknowns and 2j (or 3j) equations of statics (2D or 3D at each joint). If wo do not have enough equations of statics then the problem is indeterminate, if we have too many equations then the truss is unstable.

	2D	3D
	Static Indeterminacy	
External	$R > 3$	$R > 6$
Unstable	$M + R < 2j$	$M + R < 3j$

Classification of Trusses



Where, D_s - Degree of static indeterminacy

Assumption Involved in Analysis of Trusses

- All the joints are pin connected and free from friction.
- Members will be subjected to only axial forces.
- Self wt. of members is negligible.
- The loading is such that force in the member are within elastic limit.

Procedure of Analysis

- Find degree of static indeterminacy using $D_s = m + r_e - 2j$
  m = No. of members; r = No. of external reasons; j = No. of joints
  If $D_s = 0$, Truss is determinate and stable.

Determinate Truss can be Analyzed by Methods

(a) Method of joints
(b) Method of sections
(c) Graphical method or Williot mohr diagram

- If $D_s > 0$, then Truss is Indeterminate
  Indeterminate truss can be analyzed by methods
  (a) Unit load method; (b) Maxwell's method ; (c) Graphical method

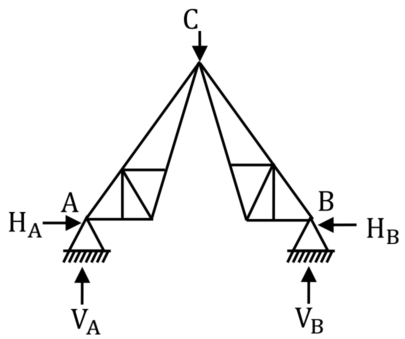
Analysis of Determinate Trusses

Methods of Joints

1. Every joints there are two equation of equilibrium i.e., $\sum F_x = 0$ and $\sum F_y = 0$
   This method is not applicable for numbers of unknowns at any point are more than 2
2. In order to find internal reactions, following equations equilibrium may be used
   $\sum F_x = 0, \sum F_y = 0, \sum M_Z = 0$
3. Not more than three equilibrium in general cannot be obtained until and unless additional equations of equilibrium are provided.
4. After finding the external equations, apply joint equilibrium at each joint one by one.
5. Tensile force are assumed to be (+) and compressive force are assumed to be (−)

Notes: If truss is externally indeterminate by degree 1 but internally, determinate by−1, then $D_s = 0$. In such case in order to find external reactions, there must be conditions such that a part of truss may be rotated about a practical joint.

Example:



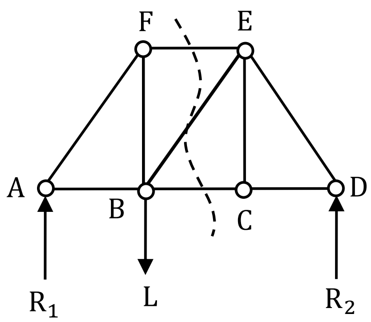
$D_s = m + r - 2j = 18 + 4 - 2 \times 11 = 0$
$D_{se} = 4 - 3 = 1$
$D_{si} = -1$
No. of equilibrium conditions = 3 + Condition obtained through rotation at C i.e., $\sum M_c = 0$

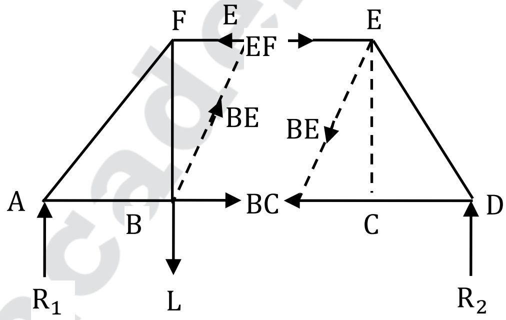
1. If at a joint three members meet and two of them are collinear and there is no external force at that joint, then the third members carries zero force always.
2. If at a joint only two members meet and there is no external force at that joint and if members are not collinearly then both members will carry zero forces.

Method of Sections

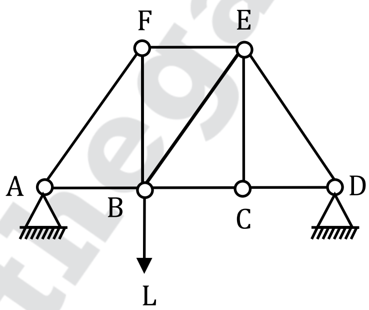
Take advantage of the 3rd or moment equation of equilibrium by selecting an entire section of truss

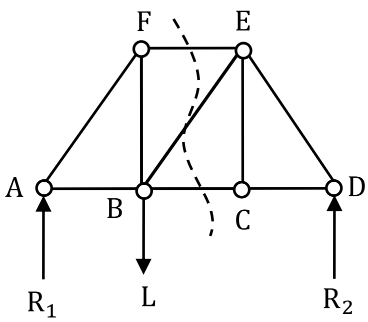
- Equilibrium under non-concurrent force system.
- Not more than 3 members whose force are unknown should be cut in a single section since we have only 3 independent equilibrium equations.





1. Find out the reactions from equilibrium of whole truss
2. To find force in member BE:
3. Cut an imaginary section (dotted line)
4. Each side of the truss section should remain in equilibrium





- Apply to each out member the force exerted on it by the member cut away.
  The left hand sections is in equilibrium under L, R_1, BC, BE and EF
- Draw the forces with proper senses (else assume)
- Moment @B → EF
- $L > R_1; \rightarrow \Sigma_y = 0 \rightarrow BE$
- Moment @ E and observation of whole truss → BC
- Forces acting towards cut section → Compressive
- Forces acting away from the cut section → Tensile